

Rational Equations

An Equation has an equal sign

An Expression does not have an equal sign

To solve equations containing rational expressions

Step 1: Clear the equation of fractions by multiplying EACH term (expression) on both sides of the equation by the LCD of all rational expressions.

Step 2: Solve the equation.

Examples:

$$\frac{x+4}{3x} + \frac{x+8}{5x} = 2$$

$$\frac{x-1}{x-5} = \frac{4}{x-5}$$

$$\frac{x^2}{x-3} = \frac{9}{x-3}$$

$$\frac{2}{x+5} + \frac{1}{x-5} = \frac{16}{x^2-25}$$

Let $f(x) = x + \frac{6}{x}$ Find all values of a for which $f(a) = 5$.

Extraneous Solutions – When a proposed solution makes the denominator 0 it must be rejected as a solution of the original equation. Also, if after checking the solution, a solution does not make the equation true you must reject the solution. When solving rational equations, be sure to list any restrictions as your first step.

Solving an equation containing rational expressions

Step 1: Multiply both sides of the equation by the LCD

Step 2: Simplify both sides of the equation

Step 3: Determine if the equation is linear, quadratic, or higher and solve accordingly

Step 4: Check the solution in the original equation

Solving Applications Using Rational Equations

Steps for solving:

Step 1: Familiarize.

Step 2: Translate

Step 3 Carry out (solve the problem)

Solving Equations for a Specified Variable

- 1: Clear Equations of fractions or rational expressions by multiplying by the LCD
- 2: Use the distributive property to remove grouping symbols
- 3: combine like terms
- 4: Use the addition property of equality
- 5: Use the distributive property and multiplication property of equality to get the specified variable alone.

Step 4: Check

Step 5: State the solution correctly. Explain answer and label if needed.

Examples:

The reciprocal of 10, plus the reciprocal of 15, is the reciprocal of what number?

The sum of a number and 21 times its reciprocal is -10. Find the number.

Filling a tank:

A water tank can be filled in 18 hours by the town office well alone; the tank can be filled in 22 hours by the high school well alone. How long will it take to fill the tank if both wells are working?

Hotel Management: The AH 400 can purify the air in a conference hall in 15 fewer minutes than it takes the A750 Air Purifier to do the same job. Together the two machines can purify the air in the conference hall in 10 minutes. How long would it take each machine, working alone, to purify the air in the room?

Cutting Firewood:

Joe can cut and split a cord of wood three times as fast as Sam can. When they work together, it takes them 5 hours. How long would it take each of them to do the job alone?

Painting a car

It takes Mary 9 hours longer than Zoe to paint a car. Working together, they can do the job in 20 hours. How long would it take each, working alone, to paint a car?

Boating:

The current in the river moves at a rate of 4 mph. Alex's boat motors 6 mi upstream in the same time it takes to motor 12 mi downstream. What is the speed of the boat in still water?

Train speed:

The speed of the freight train is 18 mph less than the speed of the passenger train. The passenger train travels 500 mi in the same time that the freight train travels 300 mi. Find the speed of each train.

Shipping

A tugboat goes 10 mph in still water. It travels 24 km upriver and 24 km downriver in a total time of 5 hours. What is the speed of the current?